KENNEDY SCHOOL
The Complete Child

Kennedy Maths Olympiad 2008

CLASS IX – KEY

1.	C	16.	D	31.	D	46.	A	61.	B,C
2.	A	17.	C	32.	A	47.	B	62.	A,C,D
3.	C	18.	D	33.	D	48.	D	63.	A,B,C
4.	C	19.	B	34.	A	49.	C	64.	C,D
5.	B	20.	C	35.	C	50.	D	65.	B,C
								66.	-3600
6.	A	21.	D	36.	C	51.	A	67.	55
7.	D	22.	B	37.	B	52.	D	68.	825 (or) 518
8.	C	23.	C	38.	B	53.	D	69.	2036
9.	A	24.	B	39.	D	54.	A	70.	105^{o}
10.	C	25.	C	40.	C	55.	A	71.	2
11.	D	26.	A	41.	A	56.	A,B,C,D	72.	600/19 (Or) $31\frac{11}{19}$ (or) 31.5%
12.	C	27.	D	42.	C	57.	A,D	73.	7.5
13.	C	28.	B	43.	C	58.	A,C	74.	$256^{0.88}$ (or) $2^{7.04}$
14.	C	29.	C	44.	C	59.	A,C,B,D	75.	$4\sqrt{3}:9$ (or) $\sqrt{3}:\frac{9}{4}$ (or) 6.92:9
15.	A	30.	D	45.	C	60.	B,C,D		

CLASS – IX

SOLUTIONS

01. **C**

$x = 7 + 4\sqrt{3} \Rightarrow \frac{1}{x} = \frac{1}{7+4\sqrt{3}} = \frac{7-4\sqrt{3}}{49-48} = 7-4\sqrt{3}$

$= x + \frac{1}{x} = 7+4\sqrt{3}+7-4\sqrt{3} = 14$ $= y = 7 - 4\sqrt{3} \Rightarrow \frac{1}{y} = \frac{1}{7-4\sqrt{3}} =$

$\frac{7+4\sqrt{3}}{49-48} = 7+4\sqrt{3}$

$= y - \frac{1}{y} = 7-4\sqrt{3}-7-4\sqrt{3} = -8\sqrt{3}$

$= \frac{x+\frac{1}{x}}{y-\frac{1}{y}} = \frac{14}{-8\sqrt{3}} = \frac{-7}{4\sqrt{3}}$

02. **A**

$= \log\left(\frac{x+y}{3}\right) = \log\sqrt{xy} \Rightarrow \frac{x+y}{3} = \sqrt{xy} \Rightarrow (x+y)^2 = 9xy$

$= x^2 + y^2 + 2xy = 9xy \Rightarrow x^2 + y^2 = 7xy \Rightarrow \frac{x}{y} + \frac{y}{x} = 7$

03. **C**

$= x^2 = yz \Rightarrow \frac{x}{y} = \frac{z}{x} = k \Rightarrow x = ky;\ z = xk = k^2y$

$= \log_{(ky)(k^2y)} ky.y^4.k^2y \times \log_{ky.y} ky.y.k^8y^4$

$= \frac{\log k^3y^6}{\log k^3y^2} \times \frac{\log k^9y^6}{\log ky^2} \Rightarrow \frac{\log(ky^2)^3}{\log k^3y^2} \times \frac{\log(k^3y^2)^3}{\log ky^2}$

$= \frac{3\log(ky^2)}{\log k^3y^2} \times \frac{3\log k^3y^2}{\log ky^2} = 9$

04. **C** 2 x 2, 2 x 1

05. **B** $a:b:c = 1:1:\sqrt{2}$

$\frac{p}{c} = \frac{1+1+\sqrt{2}}{\sqrt{2}} = \frac{2+\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}+1}{1}$

06. **A**

Square of any number is positive.

Sum of positive numbers $\neq$ 0

But $(a-1)^2 + (b-2)^2 + (c-3)^2 + (d-4)^2 = 0$ means each term is zero.

$\therefore$ a = 1, b = 2, c = 3 and d = 4

abcd+1 = 1 x 2 x 3 x 4 + 1 = 25

07. **D** $\frac{1}{1+x^a} + \frac{1}{1+x^b} = 1$

$= 1 + x^b + 1 + x^a = (1+x^a)(1+x^b)$

$= 1 + x^b + 1 + x^a = 1 + x^b + x^a + x^{a+b} \Rightarrow x^{a+b} = 1 = x^0$

$\therefore a + b = 0$

08. **C** $\{(2,48), (4,42), (6,32), (8,18)\}$

09. **A** $A \cap B = \varphi$

10. **C** A (0 ,4) B (3, 0) C (3, 5)

$AB^2 = 25$ $BC^2 = 25$ $CA^2 = 10$

$AB^2 + BC^2 > CA^2$ (Acute)

$AB^2 + BC^2 = CA^2$ (Right)

$AB^2 + BC^2 < CA^2$ (Obtuse)

11. **D**

Such a triangle does not exist

$h_1 : h_2 : h_3 = 1 : 2 : 3 \qquad h_1 = x \quad h_2 = 2x \quad h^3 = 3x$

$\Delta = \frac{1}{2} b_1 x = \frac{1}{2} b_2 2x = \frac{1}{2} b_3 .3x \Rightarrow b_1 = 2b_2 = 3b_3$

$b_1 = 2b_2 = 3b_2 = k\ ,\ b_1 = k,\ b_2 = \frac{k}{2}, b_3 = \frac{k}{3}$

$b_1 : b_2 : b_3 = k : \frac{k}{2} : \frac{k}{3} = 1 : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2$

$b_1 = 6y \quad b_2 = 3y \quad b_3 = 2y \quad b_2 + b_3 \ngeq b_1$

12. **C**

$x^3 - x^2 - 2x + 2 = 0 \Rightarrow x^2(x-1) - 2(x-1) = 0$

$(x-1)(x^2 - 2) = 0 \Rightarrow x = 1,\ \sqrt{2}\ ,\ -\sqrt{2}$

$(1)^4 + \left(\sqrt{2}\right)^4 + \left(-\sqrt{2}\right)^4 = 1 + 4 + 4 = 9$

13. **C**

$\angle A + \angle B + \angle C = 180$

$\frac{\angle A}{2} + \frac{\angle B}{2} + \frac{\angle C}{2} = 90$

$\angle OAB + \angle OBC + \angle OCA = 90$

14. **C**

2 x 3 x 5 . . . . . = ……. 0

15. **A**

$100^{30} - 20$

$10^{60} - 20$

10000 . . . . . 00 (60 zeroes) (61 places)

$\downarrow \qquad\qquad 20$

9 . . . . . 99980 (60 places)

One zero -	0	→	0
One eight -	8	→	8
58 nines -	58x9	→	522
			530

16. **D**

$d = \sqrt{(4-2)^2 + (1-3)^2} = \sqrt{4+4} = 2\sqrt{2}$

$A = \frac{d^2}{2} = \frac{8}{2} = 4$

A (a, 0) B (0, 6) C (1, 1)

17. **C**

Slope of AB = slope of BC

$\frac{b-0}{0-a} = \frac{1-b}{1-0} \Rightarrow \frac{b}{-a} = \frac{1-b}{1} \Rightarrow b = -a + ab$

$a + b = ab$

18. **D** Need not A = 0 or B = 0

19. **B** Parallelogram

20. **C** $(A \cap B) - B$

21. **D** $\begin{pmatrix}2 & -3 & 4\end{pmatrix}\begin{pmatrix}3\\2\\1\end{pmatrix} + \begin{pmatrix}0 & 2 & 3\end{pmatrix}\begin{pmatrix}2\\2\\4\end{pmatrix}$

= (6 -6 + 4) + (0 + 4 + 12) = (4) + (16) = (20)

22. **B** $(1+2i)^2 - 2(1+2i) + k = 0 \Rightarrow 1 + 4i^2 + 4i - 2 - 4i + k = 0$

1 – 4 – 2 + k = 0, k = 5

23. **C**

$\alpha + \beta = \frac{-b}{a}, \quad \alpha\beta = \frac{c}{a}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \Rightarrow \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$

But $\alpha + \beta = \alpha^2 + \beta^2$

$\frac{-b}{a} = \frac{b^2 - 2ac}{a^2} \Rightarrow -ab = b^2 - 2ac$

$b^2 + ab = 2ac \Rightarrow b(a + b) = 2ac$

24. **B**

$2^{x-2} = 5^{2-x} \Rightarrow 2^{x-2} = \frac{1}{5^{x-2}} \Rightarrow 2^{x-2} \times 5^{x-2} = 1$

$(10)^{x-2} = 10^o \Rightarrow x - 2 = 0, \quad x = 2$

25. **C**

$2008 = \sqrt{x + \sqrt{x + \sqrt{x + }}} \qquad \Rightarrow \; 2008 = \sqrt{x + 2008}$

$(2008)^2 = x + 2008$

$\Rightarrow \quad x = (2008)^2 - (2008)$

$= (2008)\,(2008 - 1)$

$= 2008 \times 2007$

26. **A**

D is circumcentre

$\therefore$ AD = DC = BD

$\therefore$ AD = DC = BD = AB

$\therefore$ In ΔABD

AB = BD = AD

$\therefore \quad \angle A = 60^o$

27. **D**

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 0$

$x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0$

$\dfrac{x^2 + y^2 + z^2}{xy + yz + zx} = -2$

28. **B** (a, b)

29. **C** 999 $1^3, 2^3, 3^3, \ldots 10^3$ Range $= 10^3 - 1^3 = 1000 - 1 = 999$

30. **D** Sectors

31. **D** None

32. **A**

$A \cap B = \varphi$ then $n(A \cup B) = n(A) + n(B) = 9 + 13 = 22$

$A \subset B, \quad n(A \cap B) = 9 \Rightarrow n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 9 + 13 - 9 = 13$

33. **D**

$$\frac{\text{cube}}{\text{sphere}} = \frac{a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^2} = \frac{3a^3}{4\pi \times \frac{a^3}{8}} = \frac{6}{\pi} = 6 : \pi$$

34. **A**

$r = \sqrt{(5+7)^2 + (5)^2} = \sqrt{144 + 25} = 13$

$\pi r^2 = 169\pi$

35. **C**

$25^{1/3} - 25^{1/3} \Rightarrow a^2 + ab + b^2 = \left(25^{1/3}\right)^2 + 25^{1/3}.25^{-1/3} + \left(25^{-1/3}\right)^2$

$= \left(5^{2/3}\right)^2 + 1 + \left(5^{-2/3}\right)^2 = 5^{4/3} + 1 + 5^{-4/3}$

36. **C**

$\log_{abc} ab + \log_{abc} bc + \log_{abc} ca = \log_{abc} (abc)^2 = 2$

37. **B**

$n(A \cup B \cup C) =$ n(A) + n(B) + n(C)

$-n(A \cap B) - n(B \cap C)$

$-n(C \cap A) + n(A \cap B \cap C)$

$\Rightarrow \quad 31 = 20 + 16 + 8 - 3 - x - 4 - x - 2 - x + x$

$\Rightarrow \quad 2x = 44 - 40 = 4$

$\therefore \; x = 2$

38. **B**

$\sqrt{11 + 2\sqrt{30}} = \sqrt{6} + \sqrt{5} \Rightarrow x^2 + y^2 = 36 + 25 = 61$

39. **D**

$a + b + c = 58 \Rightarrow a + b = 58 - 24 = 34$

$a^2 + b^2 = 24^2 = 576$

$(a + b)^2 = (34)^2 \quad \Rightarrow \quad a^2 + b^2 + 2ab = 1156$

$2ab = 1156 - 516 = 580 \; \Rightarrow \; \frac{2ab}{4} = \frac{580}{4} = 145$

40. **C**

Area of shaded part = $\frac{\pi r^2}{2} = 308 \quad \Rightarrow \quad \pi r^2 = 616$

$r^2 = 616 \times \frac{7}{22} = 196 \quad \Rightarrow \quad r = 14$

$2\pi r = 2 \times \frac{22}{7} \times 14 = 88$ cm

88 cm = $\frac{88}{100}$ m = 0.88 mts.

41. **A**

Area ($\square$ABCD) = 15 x 6 = 90 sq.cm.

Area of circle = $\pi r^2 = \frac{22}{7} \times 3^2 = \frac{198}{7}$

2 circles + 1 semi circle = $2 \times \frac{198}{7} + \frac{198}{2 \times 7} = \frac{198}{7} \times \frac{5}{2}$

$\frac{99\times5}{7}=\frac{495}{7}$

90 - $\frac{495}{7}=\frac{135}{7}$

5 equal parts area = $\frac{135}{7}$ 1 part = $\frac{135}{7\times5}=\frac{27}{7}$

4 parts = $\frac{4\times27}{7}=\frac{108}{7}$

42. **C**

$$\frac{\text{Sphere volume}}{\text{Cone volume}}=\frac{\frac{4}{3}\pi(10.5)^3}{\frac{1}{3}\pi(3.5)^2\times3}=\frac{4\times10.5\times10.5\times10.5}{3\times3.5\times3.5}$$

$$=\frac{4630.5}{36.75}=126$$

43. **C** y = mx

44. **C** $x^4=14641=11^4\Rightarrow x=11$ Ans : 2 digits

45. **C** $\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\ .\ .\ .\ .\ .\ .\ .\ .+\frac{1}{\sqrt{100}+\sqrt{99}}$

46. **A**

$$\sum_{n=1}^{100}\left(\frac{1}{n}\right)^{-1}=\left(\frac{1}{1}\right)^{-1}+\left(\frac{1}{2}\right)^{-1}+\left(\frac{1}{3}\right)^{-1}+\ .\ .\ .\ .\ .\ +\left(\frac{1}{100}\right)^{-1}$$

$$=1+2+3+\ .\ .\ .\ .+100$$

$$=\frac{n(n+1)}{2}=\frac{100(100+1)}{2}=5050$$

47. **B**

P = 3 + 4 + 5 = 12

A = $\frac{1}{2}\times3\times4=6$

P = 2A

48. **D** None

49. **C**

$\Delta ADR \sim \Delta CQR$

(ASA)

$\Rightarrow\quad x^2=(7-x)^2+3^2$

$\Rightarrow \quad x^2 = 40 - 14x + x^2 + 9$

$\Rightarrow \quad 14x = 58, \quad x = \frac{58}{14} = \frac{29}{7}$

Shaded area = b x h = $\frac{29}{7} \times 3 = \frac{87}{7}$

50. **D** 

51. **A** Additive inverse of A

52. **D** Sphere

53. **D** $(A \cup B)' = A' \cap B' \quad \Rightarrow \quad (A' \cap B')' = (A \cup B)' = A \cup B$

54. **A** Rhombus

55. **A** In five movements → 365^o 5^o excess

$72 \times 5 = 360^o$

$\therefore$ 72 movements

56. **A, B, C, D**

57. **A, D**

58. **A, C**

59. **A, C, B, D** 2, 4, 16

60. **B, C, D**

61. **B, C**

$3^{1+x} + 3^{1-x} = 10 \quad \Rightarrow \quad 3 \times 3^x + \frac{3}{3^x} = 10$

$= 3a + \frac{3}{a} = 10$

$\Rightarrow \quad 3a^2 + 3 = 10a$

$3a^2 - 10a + 3 = 0$

$\Rightarrow \quad 3a^2 - 9a - a + 3 = 0$

$3a(a-3) - 1(a-3) = 0 \quad \Rightarrow \quad (a-3)(3a-1) = 0$

$a = 3$ or $1/3 \quad \Rightarrow \quad 3^x = 3^1$ or 3^{-1}

62. **A, C, D**

$\Rightarrow AQ^2 + BP^2 = \frac{a^2}{4} + b^2 + a^2 + \frac{b^2}{4}$

$= \frac{5}{4}(a^2 + b^2)$

$= \frac{5}{4}(AB^2)$

$\Rightarrow AB^2 + PQ^2 = C^2 + \frac{a^2}{4} + \frac{b^2}{4} = a^2 + b^2 + \frac{a^2}{4} + \frac{b^2}{4} = \frac{5}{4}(AB^2)$

$\Rightarrow \frac{5}{4}(AB^2) = \frac{5}{4}(2PQ)^2$ $\quad PQ = \frac{1}{2}AB$

$= 5PQ^2$

63. **A, B, C**

64. **C, D** If diagonals perpendicular = $\frac{1}{2}d_1d_2$

$\frac{1}{2}(h_1 + h^2)$

65. **B, C** $(A+B)^T = A^T + B^T$

$(AB)^T = B^T A^T$

66. **-3600** The pattern is x – 1, x – 2, x – 3, x – 4, x – 5, x – 6
So, 600 x (-6) = -3600

67. **55** Adding $1^2, 2^2, 3^2, 4^2, 5^2, 6^2$ So 31+25 should be 56.

68. **518** All other numbers are odd.
825 Sum of all digits is odd for other numbers.

69. **2036** Since this is a leap year, the same calendar repeats after 4 x 7 = 28 years

70. 105^o

71. **2** Unit's digit in 2^x is either 2, 4, 8, 6. Hence, 2^{65} is having a unit's digit 2. The remainder is 2

72. $\frac{600}{19}$ **(or)** $31\frac{11}{19}$ **(or) 31.58%** $\quad \left(\frac{24}{(100-24)} \times 100\right)\%$

73. **7.5** $\frac{x}{3} - \frac{x}{3.75} = \frac{1}{2} \Rightarrow x = \frac{3 \times 3.75}{2 \times 0.75} = 7.5$

74. $\mathbf{256^{0.88}}$ (or) $\mathbf{2^{7.04}}$ $\quad (256)^{0.18+0.7} = (256)^{0.88} = (2^8)^{0.88} = 2^{7.04}$

75. $4\sqrt{3} : 9$ **(or)** $\sqrt{3} : \frac{9}{4}$ **(or)** $6.92 : 9$

Let perimeter of equilateral triangle be 3x

$\therefore$ Area = $\frac{\sqrt{3}}{4}x^2$

Area of a square is $\left(\frac{3}{4}x\right)^2 = \frac{9}{16}x^2$

$\therefore$ Ratio of $\triangle : \square$ = $\frac{\sqrt{3}}{4}x^2 : \frac{9}{16}x^2$

= $4\sqrt{3} : 9$ (or) $\sqrt{3} : \frac{9}{4}$ (or) $6.92 : 9$