KENNEDY SCHOOL
The Complete Child

Kennedy Maths Olympiad 2008

CLASS VII – KEY

1.	A	11.	B	21.	B	31.	D	41.	C	51.	125
2.	C	12.	B	22.	B	32.	C	42.	D	52.	-13
3.	C	13.	C	23.	C	33.	C	43.	A	53.	158
4.	C	14.	D	24.	C	34.	A	44.	C	54.	19-12-10
5.	B	15.	D	25.	C	35.	C	45.	B	55.	240^{o}
6.	D	16.	A	26.	D	36.	B	46.	D	56.	99900
7.	B	17.	D	27.	B	37.	C	47.	B	57.	300%
8.	C	18.	A	28.	C	38.	D	48.	A	58.	9.2
9.	A	19.	A	29.	A	39.	C	49.	B	59.	104
10.	A	20.	D	30.	C	40.	D	50.	D	60.	Rs. 121

CLASS – VII

SOLUTIONS

01. **A**

$a(i)^{101} = (i^2)^{50} \times i = i$

$(i^2)^{201} = (-1)^{200} \times 2^o = i$

$(i^2 - i^4)^{301} = (-1 - 1)^{300} \times i = 2^{300} \times i$

$(i^2 + i^4)^{401} = (-1 + 1)^{401} = 0$

$(i)^{101} \times (i^2)^{201} \times (i^2 - 2^4)^{301} \times (i^2 \times i^4)^{401} = i \times i \times 2^{300} \times 2 \times 0 = 0$

02. **C**

S = Midpoint of AC

B = Orthocentre

$SA = SB = SC = \frac{AC}{2} = \frac{\sqrt{16^2 + 12^2}}{2}$

$= \frac{20}{2} = 10$

03. **C**

$x - \frac{1}{x} = \sqrt{21} \quad \Rightarrow \quad x^2 + \frac{1}{x^2} - 2 = 21 \quad \Rightarrow \quad x^2 + \frac{1}{x^2} + 2 = 25 \quad \Rightarrow \quad x + \frac{1}{x} = 5$

$$\left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)=23\times5=115$$

04. **C**

$9+\sqrt{4}=11 \qquad \sqrt{9}+4=7$

x = 9, y = 4

05. **B**

$$a^b=b^a \Rightarrow \left(\frac{a}{b}\right)^{\frac{a}{b}}=\frac{a^{\left(\frac{a}{b}\right)}}{b^{\frac{a}{b}}}=\frac{a^{\frac{a}{b}}}{\left(b^a\right)^{\frac{1}{b}}}=\frac{a^{\frac{a}{b}}}{\left(a^b\right)^{\frac{1}{b}}}=\frac{a^{\frac{a}{b}}}{a}$$

06. **D**

$$0.\overline{6}+0.\overline{7}+0.\overline{3}+0.\overline{8}=\frac{6}{9}+\frac{7}{9}+\frac{3}{9}+\frac{8}{9}=\frac{24}{9}=\frac{8}{3}=2\frac{2}{3}$$

07. **B**

$a^x = b^y = c \qquad \Rightarrow \qquad a = b^{y/x}$

$a^p = b^q = d \qquad \Rightarrow \qquad a = b^{q/p}$

$$b^{\frac{y}{x}}=b^{\frac{q}{p}} \qquad \Rightarrow \qquad \frac{y}{x}=\frac{q}{p} \Rightarrow q=\frac{py}{x}$$

08. **C**

$x + yz = (x + y)(x + z) \Rightarrow x + yz = x^2 + xz + xy + yz$

$x = x^2 + xz + zy \Rightarrow x = x(x+y+z) \Rightarrow x + y + z = 1$

09. **A**

$ax^2 + bx + c = 0.x^2 + 0.x + 0 \Rightarrow a = 0,\ b = 0,\ c = 0$

$$1000x + p = 0 \Rightarrow x = \frac{-p}{1000} \Rightarrow \frac{-p}{1000}=a+b+c=0 \Rightarrow p = 0$$

10. **A**

$$\frac{x}{b+c}=\frac{y}{c+a}=\frac{c}{a+b}=k \Rightarrow x = k(b+c),\ y = k(c+a),\ z = k(a+b)$$

$(b-c)x + (c-a)y + (a-b)z \Rightarrow (b-c)k(b+c) + (c-a)k(c+a) + (a-b)k(a+b)$

$k[b^2 - c^2 + c^2 - a^2 + a^2 - b^2] = k \times 0 = 0$

11. **B**

$$\pi d - d = 30 \Rightarrow d(\pi-1)=30 \Rightarrow d\left(\frac{22}{7}-1\right)=30$$

$$\Rightarrow d=\frac{30\times7}{15}=14 \Rightarrow r = 7 \Rightarrow \pi r^2=\frac{22}{7}\times7\times7=154$$

12. **B**

a, b, c, d are four prime numbers

$\frac{abc}{bcd} = \frac{a}{d} = \frac{385}{1001} = \frac{5}{13} \Rightarrow a = 5,\ d = 13$

13. **C**

$c + d + 6 = a - b$

$(a - b) - (c + d) = 6$

$a - b - c - d = 6$

$-a - b + c - d = 3$

$2a - 2c = 3 \qquad (a - c) = 1.5$

14. **D**

$ab^2c^3 = 2^3 \qquad a^2b = 4^3$

$(ab^2c^3) \times (a^2 b) = 2^3 \times (2^2)^3$

$a^3 b^3 c^3 = 2^3 \times 2^6 \Rightarrow (abc)^3 = 2^9 = (2^3)^3$

$abc = 8$

15. **D**

$a = 3,\ b = 5,\ c = 7$

$$\frac{\left(b + \sqrt{a}\right)\left(b - \sqrt{a}\right)}{\left(c - \sqrt{b}\right)\left(c + \sqrt{b}\right)} = \frac{\left(5 + \sqrt{3}\right)\left(5 - \sqrt{3}\right)}{\left(7 - \sqrt{5}\right)\left(7 + \sqrt{5}\right)} = \frac{25 - 3}{49 - 5} = \frac{22}{44} = \frac{1}{2}$$

16. **A**

$1 < 3 < 4 \Rightarrow \sqrt{1} < \sqrt{3} < \sqrt{4} \Rightarrow 1 < \sqrt{3} < 2$

or $\sqrt{3} = 1.732$

17. **D** None

18. **A**

AG : GD = (2x + 2) : 3x – 7 = 2 : 1

$2x + 2 = 2(3x - 7)$

$6x - 14 = 2x + 2$

$4x = 16; \qquad x = 14$

19. **A**

$1 + ab + a + a^2b = (1+a)\,k \Rightarrow (1+a) + ab\,(1+a) = (1+a)k$

$(1+a)\,(1+ab) = (1+a)\,k \Rightarrow k = 1 + ab$

$(k+1)^2 - (k-1)^2 = 4k \Rightarrow 4\,(1+ab)$

20. **D**

$2\,(l + b) = 16x^3 - 6x^2 + 12x + 4$

$l + b = 8x^3 - 3x^2 + 6x + 2$

$8x^2 + 3x + b = 8x^3 - 3x^2 + 6x + 2$

$b = 8x^3 - 11x^2 + 3x + 2$

21. **B**

$(x+1)^2 = 4 \Rightarrow x+1 = \pm 2 \Rightarrow x + 1 = 2;\ x = 1$

$x + 1 = -2;\ x = -3$

$(y+2)^2 = 9 \Rightarrow y + 2 = \pm 3 \Rightarrow y + 2 = 3;\ y = 1$

$y + 2 = -3;\ y = -5$

$xy = (-3) \times (-5) = 15$

22. **B**

$$\frac{11+22+33+44+55+66+77+88+99}{9} = \frac{495}{9} = 55$$

23. **C**

No. of sections = No. of students per section = a

Total No. of students = $a^2 = 9x^2 - 24x + 16 = (3x)^2 - 2(3x)\,4 + 4^2$

$a^2 = (3x-4)^2 \Rightarrow a = 3x - 4$

24. **C**

$$\sqrt{2^m \times 3^n} = 72 \Rightarrow 2^m \times 3^n = (72)^2 = \left(2^3 \times 3^2\right)^2 = 2^6 \times 3^4$$

$m = 6,\ n = 4 \Rightarrow m+n = 10$

25. **C**

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^2 + b^2 - 2ab = (a-b)(a-b)$$

$$a^2 + b^2 + 2ab = (a+b)(a+b)$$

26. **D** $\quad V = \frac{4}{3}\pi r^2 h \Rightarrow r^2 = \frac{3v}{4\pi h} \Rightarrow r = \sqrt{\frac{3v}{4\pi h}}$

27. **B**

$x + y = \theta$

$\theta - z = 60$

$x + y - z = 60$

$(y + z) + y - z = 60$

$2y = 60$

$y = 30$

28. **C** A & B

29. **A** $\sqrt{1+\sqrt{4+\sqrt{16+\sqrt{81}}}} = \sqrt{1+\sqrt{4+\sqrt{16+9}}} = \sqrt{1+\sqrt{4+5}} = \sqrt{1+3} = 2$

30. **C**

$\angle BAD = 45^o$

$\angle BCE = 45^o$

In square BDOE, 45 + 90 + 90 +x = 360

x = 135

$\therefore \angle AOC = 135^o$

31. **D**

$\angle AOC = 135^o$

$x = \frac{45}{2} = 22\frac{1}{2}$

$\angle A = 67\frac{1}{2}$ $\angle B = 45$ $\angle C = 67\frac{1}{2}$

Acute & Isosceles

32. **C** 360^o

33. **C** Right

34. **A** Left

35. **C** $7.3\overline{58}$

36. **B** Negative

37. **C** 1

38. **D** All of the above

39. **C** Semicircle

40. **D** l + 2t, b+ 2t, h + t

41. **C** $a = \frac{p-m}{p+m} \Rightarrow a(p+m) = p - m \Rightarrow ap + am = p - m$

42. **D**

$S_1^3 + S_2^3 + S_3^3 = 1^3 + 6^3 + 8^3 = 1 + 216 + 512 = 729 = 9^3$

$S^3 = 9^3 \Rightarrow S = 9$

L.S.A. $4\,l^2 = 4 \times 9^2 = 324$ Sq.cm.

43. **A**

In ΔABC, AD = 21 AD : Altitude (median)

AG : GD = 2 :1 O : Centroid (Orthocentre)

$AG = \frac{2}{3} \times 21 = 14$ cm (Circum radius)

$GD = \frac{1}{3} \times 21 = 7$ cm (Inradius)

$\pi R^2 - \pi r^2 = \pi(14+7)(14-7) = \frac{22}{7} \times 21 \times 7 = 462$ sq.cm.

44. **C** Does not exist

45. **B** 1 $\frac{0.80}{80} \times 100 \Rightarrow \frac{80}{80} = 1$

46. **D** Circumference

47. **B** ac > bc

48. **A** Three sides, Three angles

49. **B** Isosceles trapezium

50. **D** None

51. **125** The pattern is 1^3, 2^3, 3^3, 4^3, 5^3

52. **-13** The pattern is +1, +2, +3, +4, +5

53. **158** All are multiples of 3 except 158

54. **19-12-10** 19 December, 2010

55. **240°** 240^o

56. **99900** 99900

57. **300%** Let perimeter be 4x; increased by 100% → 8x

Side → x, increased by 100% → 2x

Area → x^2, increased by 100% → $(2x)^2 = 4x^2$

Increase = 300%

58. **9.2** 6 x 7 = 42; (6+5) x 8 = 88 $\frac{88-42}{5}=\frac{46}{5}=9.2$

59. **104** $\frac{128}{10}\times 13=104$

60. **Rs. 121** $\frac{99}{(100-10)}\times 100=\text{Rs.}110 \rightarrow 110\times\frac{(100+10)}{100}=\text{Rs. }121$