KENNEDY SCHOOL
The Complete Child

Kennedy Maths Olympiad 2008

CLASS VIII – KEY

1.	B	16.	B	31.	C	46.	C	61.	A,B
2.	B	17.	C	32.	A	47.	B	62.	B,C,D
3.	C	18.	C	33.	D	48.	C	63.	A,C
4.	D	19.	A	34.	B	49.	A	64.	A,B
5.	A	20.	A	35.	C	50.	D	65.	A,B,C
6.	C	21.	C	36.	D	51.	D	66.	625
7.	D	22.	A	37.	B	52.	C	67.	30
8.	A	23.	B	38.	A	53.	D	68.	225
9.	D	24.	C	39.	B	54.	A	69.	Wednesday
10.	B	25.	B	40.	C	55.	D	70.	44
11.	C	26.	D	41.	C	56.	A,B,C	71.	35
12.	A	27.	D	42.	B	57.	A,B,C	72.	2:3
13.	C	28.	B	43.	A	58.	A,B	73.	15%
14.	C	29.	A	44.	D	59.	A,C	74.	18 Years
15.	B	30.	A	45.	A	60.	B,C	75.	8

CLASS – VIII

SOLUTIONS

01. **B**

$n(A - B) = 25 + x$

$n(B - A) = 2x$

$n(A \cap B) = 2x$

$n(A) = n(A - B) + n(A \cap B) = 25 + 3x$

$n(B) = n(B - A) + n(A \cap B) = 4x$

$n(A) = 2\, n(B)$

$25 + 3x = 2(4x)$

$8x = 25 + 3x$

$5x = 25$

$x = 5$

02. **B** Equal Sets

03. **C**

$6x + y = 21,\ x : y = 1 : 3 \Rightarrow \frac{x}{y} = \frac{1}{3} \quad \Rightarrow \quad x = \frac{y}{3}$

$6 \times \frac{y}{3} + y = 21$

$3y = 21 \quad \Rightarrow \quad y = 7$

04. **D**

ar ($\square$ ABCD) = 12 x 12 = 144

ar (ΔAEF) = $\frac{1}{2} \times 5 \times 7 = \frac{35}{2}$

ar (ΔFBC) = $\frac{1}{2} \times 5 \times 12 = 30$

ar $(\Delta DEG) = \frac{1}{2} \times 7 \times 8 = 28$

144 - $\left(\frac{35}{2} + 30 + 28\right) = 144 - \left(\frac{35 + 60 + 56}{2}\right) = 144 - 75.5 = 68.5$

05. **A**

$A \subset B$

If $A \subset B$ then A – B = φ

$A - (A - B) = A - \varphi = A$

06. **C**

$f(x) = 5x^3 + 6x^2 + 3x + 5m$

$g(x) = 7x^2 + 8x - 19$

$f(-2) = g(-2)$

$5(-2)^3 + 6(-2)^2 + 3(-2) + 5m = 7(-2)^2 + 8(-2) - 19$

$-40 + 24 - 6 + 5m = 28 - 16 - 19$

$5m = 15$

$m = 3$

07. **D**

$a^m\ a^n = a^{mn} \quad \Rightarrow \quad a^{m+n} = a^{mn} \quad \Rightarrow \quad m+n = mn$

$m(n-2) + n(m-2) \Rightarrow \quad mn - 2m + mn - 2n$

$2mn - 2(m+n) = 2mn - 2mn = 0$

08. **A**

$$\frac{x-a^2b}{ab^2}+\frac{x-ab^2}{a^2b}=2 \Rightarrow \frac{a\left(x-a^2b\right)+b\left(x-ab^2\right)}{a^2b^2}=2$$

$ax - a^3b + bx - ab^3 = 2a^2b^2 \Rightarrow (a+b)x = 2a^2b^2 + a^3b + ab^3$

$$(a+b)x = ab\,(2ab + a^2 + b^2) \Rightarrow x=\frac{ab\left(a+b\right)^2}{a+b}=ab\left(a+b\right)$$

09. **D**

Let the number be x

$$10\%\text{ of } x=\frac{x}{10}$$

$$\text{Resulting number} = \frac{11x}{10}$$

$$20\%\text{ of } \frac{11x}{10}=\frac{20}{100}\times\frac{11x}{10}=\frac{11x}{50}$$

$$\text{New number} = \frac{11x}{10}+\frac{11x}{50}=\frac{66x}{50}=\frac{33x}{25}$$

$$\text{Increased percentage} = \frac{\frac{33x}{25}-x}{x}=100=\frac{8x}{25x}\times100=32$$

10. **B**

A = B =

$B' =$

$A \cup B' =$

$\left(A \cup B'\right) \cap B =$

$\therefore A \cap B$

11. **C**

A = {(3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73)}

n(A) = 8

Proper Subsets = $2^n - 1 = 2^8 - 1 = 255$

12. **A**

R = {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}

n(R) = 6

13. **C**

ar (llgm AECD) + ar (ΔBCE)

$10 \times 12 + \frac{1}{2} \times 10 \times 12;\ \ 120 + 60 = 180$

14. **C**

No. of rows = 4 No. of columns = 500

Length = 502 x 4 = 2008 Breadth = 4 x 4 = 16

Perimeter = 2(l+b) = 2 (2008+16) = 2(2024) = 4048 cm

15. **B**

$256 = 2^8$, $2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^8$

$2^5 = 32$

16. **B**

$$\frac{x}{y+1}=\frac{6x}{6y+6};\ \frac{2x}{2y-1}=\frac{6x}{6y-3};\ \frac{2x}{2y+1}=\frac{6x}{6y+3};\ \frac{3x}{3y+1}=\frac{6x}{6y+2}$$

$6y - 3 < 6y + 2 < 6y + 3 < 6y + 6$

$$\frac{6x}{6y-3}>\frac{6x}{6y+2}>\frac{6x}{6y+3}>\frac{6x}{6y+6}$$ $\frac{2x}{2y-1}$ is greater

17. **C**

$n^{200} < 6^{300}$

$\left(n^2\right)^{100} < \left(6^3\right)^{100}$

$n^2 < 216 \Rightarrow 14^2 < 216$

$|\therefore n = 14$

18. **C**

$$\frac{3}{12}=\frac{1}{4}=25\%$$

19. **A**

a – 2005 = b + 2006 = c – 2007 = d + 2008 = k

a = k + 2005, b = k – 2006, c = k + 2007

d = k – 2008

d < b < a < c

20. **A** 60

21. **C**

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ \ldots.+\frac{1}{99\times100}$$

$$\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ \ldots+\left(\frac{1}{99}-\frac{1}{100}\right)$$

$$1-\frac{1}{100}=\frac{99}{100}$$

22. **A**

2, 3, 5, 7 $n = a^2bc = 7^2 \times 5 \times 3 = 735$

$n = a^2bc = 2^2 \times 3 \times 5 = 60$

23. **B**

$2^x = 3^y = 6^x = k$

$2 = k^{\frac{1}{x}}, \quad 3 = k^{\frac{1}{y}}, \quad 6 = k^{\frac{1}{z}}$

2 x 3 = 6 $\Rightarrow \quad k^{\frac{1}{x}}.k^{\frac{1}{y}} = k^{\frac{1}{z}} \quad \Rightarrow \quad \frac{1}{x}+\frac{1}{y}=\frac{1}{z}$

$\frac{x+y}{xy}=\frac{1}{x} \quad \Rightarrow \quad x=\frac{xy}{x+y}$

24. **C**

AC = 8 + 2 = 10

OD = $\frac{AC}{2}=5=OC$

OB = OC = BC = 5 – 2 = 3

In ΔBOD, $\angle B = 90$, OD = 5, OB = 3

$\therefore$ BD = $\sqrt{OD^2 - OB^2} = \sqrt{25-9} = 4$

25. **B**

$70 + \angle B + \angle C = 180^o$

$\angle B + \angle C = 110^o$; $\qquad \frac{1}{2}\angle B + \frac{1}{2}\angle C = 55^o$

$\Delta BOC, \ \angle BOC + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 180^o$

$\angle BOC = 180 - 55 = 125^o$

26. **D**

n/mm then mn – 1 divided by n, remainder is n – 1

Ex : 4/9x4 then 4 x 2 – 1 divided by 4, remainder is 4 – 1

27. **D** φ

28. **B** ax + by + c = 0

29. **A**

In ΔAOD, AD = 5, DA = 4

$\therefore$ DO = 3 cm; BD = 6 cm

$\frac{1}{2}\times AC \times BD = \frac{1}{2}\times 8 \times 6 = 24$

30. **A**

$\sqrt{2}.a = hyp$

$$a = \frac{5}{\sqrt{3}} = \frac{5\sqrt{2}}{2}$$

31. **C**

$2^{xy} = 256 = 2^{23} \Rightarrow x = 2;\ y = 3$

$(x+y)^{y-x} = 5^{3-2} = 5$

32. **A** $\sqrt{\left(\sqrt{\frac{a}{b}}\right)^2 + \left(\sqrt{\frac{b}{a}}\right)^2 - 2\left(\sqrt{\frac{a}{b}} \times \sqrt{\frac{b}{a}}\right)} = \sqrt{\left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right)^2} = \sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}$

33. **D** $a+b+c = 0 \Rightarrow a+b = -c \Rightarrow (a+b)^2 = (-c)^3$

$\Rightarrow a^3 + b^3 + 3ab(a+b) = -c^3 \Rightarrow a^3 + b^3 + c^3 = 3abc$

34. **B**

$a = 3, b = 5, c = 7, d = 11$

$i^a + i^b + i^c + i^d = i^3 + i^5 + i^7 + i^{11}$

$i^2 + i + \left(i^2\right)^2 i + \left(i^2\right)^3 i + \left(i^2\right)^5 \times i \Rightarrow -i + i - i - i = -2i$

35. **C** $2^{4-1}(2^4 - 1) = 8\ (15) = 120$ not perfect

36. **D**

Let B's cost = x

10% of x = $\frac{x}{10}$

A's cost = $\frac{11x}{10}$

A's cost + B's cost = $x + \frac{11x}{10} = \frac{21x}{10} = 2226$

$x = \frac{2226x10}{21} = 1060$

A's cost = $\frac{11x1060}{10} = 1166$

37. **B**

$a^4 + 4b^4 + 3a^2b^2 \Rightarrow \left(a^2\right)^2 + \left(2b^2\right)^2 + 2\left(a^2\left(2b^2\right)\right) - a^2b^2$

$\left(a^2 + 2b^2\right)^2 - (ab)^2 \Rightarrow \left(a^2 + 2b^2 + ab\right)\left(a^2 + 2b^2 - ab\right)$

38. **A**

$a+b+c = 0 \Rightarrow a+b = -c,\ a+c = -b,\ b+c = -a$

$$\frac{(a+b)^2}{ab}+\frac{(b+c)^2}{bc}+\frac{(c+a)^2}{ac}=\frac{c^2}{ab}+\frac{a^2}{bc}+\frac{b^2}{ac}=\frac{c^3+a^3+b^3}{abc}$$

$$\frac{3abc}{abc}=3$$

39. **B**

$$x^2+\frac{1}{x^2}=7 \quad \Rightarrow \quad x^2+\frac{1}{x^2}+2=9 \quad \Rightarrow \quad x+\frac{1}{x}=\pm 3$$

$$\left(x+\frac{1}{x}\right)^3=3^3 \quad \Rightarrow \quad x^3+\frac{1}{x^3}+3.3=27 \quad \Rightarrow \quad x^3+\frac{1}{x^3}=18$$

$$7x^3+8x+\frac{7}{x^3}+\frac{8}{x}=7\left(x^3+\frac{1}{x^3}\right)+8\left(x+\frac{1}{x}\right)=7\times 18+8\times 3=150$$

40. **C**

Let a, a+2 be two consecutive positive integers

$a^2 + (a+2)^2 = 340 \Rightarrow 2a^2 + 4a + 4 = 340$

$a^2 + 2a + 2 = 170 \Rightarrow a^2 + 2a - 168 = 0$

$(a+14)\ (a-12) = 0 \Rightarrow a = 12 \quad a+2 = 14$

41. **C**

$$\frac{360}{n}=\frac{1}{3}\left(\frac{2n-4}{n}\right)\times 90$$

$12 = 2n - 4 \Rightarrow 2n = 16 \Rightarrow 11 = 8$

42. **B** Pentagon $n=\dfrac{n(n-3)}{2} \Rightarrow n^2-3n-2n=0 \Rightarrow n = 0 \text{ or } 5$

43. **A** Rational Numbers

44. **D** $a + b = \sqrt{ab} \Rightarrow \dfrac{a}{\sqrt{ab}}+\dfrac{b}{\sqrt{ab}}=1 \Rightarrow \dfrac{\sqrt{a}}{\sqrt{b}}+\dfrac{\sqrt{b}}{\sqrt{a}}=1$

$$\frac{a}{b}+\frac{b}{a}+2=1 \Rightarrow \frac{a}{b}+\frac{b}{a}=-1$$

45. **A** $(3x-p)^2=(3x)^2-2(3x)p+p^2=9x^2-3xy$

$$6xp=3xy \Rightarrow p=\frac{3xy}{6x}=\frac{y}{2}$$

$$\frac{y}{2},\ \frac{y^2}{4}$$

46. **C** 90^o

47. **B** $(a, -b) \in Q_3$; $a<0$, $-b < 0$, $a<0$, $b>0$

48. **C** Line segment

49. **A** $$\frac{a+b}{\sqrt[3]{a}+\sqrt[3]{b}} = \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\right)\left(\left(\sqrt[3]{a}\right)^2 - \sqrt[3]{a}.\sqrt[3]{b} + \left(\sqrt[3]{b}\right)^2\right)}{\left(\sqrt[3]{a}+\sqrt[3]{b}\right)}$$

$$= \sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2}$$

50. **D** $(2+1)\ (3+1)\ (1+1) = 3 \times 4 \times 2 = 24$

51. **D** $a = 0$, $(b+c)^{(b-c)x0} = (b+c)^0 = 1$

52. **C** $\sqrt{a^2+b^2} = a+b$

53. **D** β

54. **A** Axiom

55. **D** Square

56. **A, B, C** $256 = 16^{16}$, $27 = 3^3$, $4 = 2^2$

57. **A, B, C**

58. **A, B** $a^4 - 13a^2b^2 + 36b^4 = \left(a^2\right)^2 + \left(6b^2\right)^2 - 2\left(a^2\right)\left(6b^2\right) - a^2b^2$

$$\Rightarrow \left(a^2 - 6b^2\right)^2 - (ab)^2 \Rightarrow \left(a^2 - 6b^2 + ab\right)\left(a^2 - 6b^2 - ab\right)$$

$$a^4 - 13a^2b^2 + 36b^4 = \left(a^2\right)^2 + \left(6b^2\right)^2 + 2\left(a^2\right)\left(6b^2\right) - 12a^2b^2 - 13a^2b^2$$

$$= \left(a^2 + 6b^2\right)^2 - 25a^2b^2 = \left(a^2 + 6b^2\right)^2 - (5ab)^2$$

$$= \left(a^2 + 6b^2 + 5ab\right)\left(a^2 + 6b^2 - 5ab\right)$$

59. **A, C** $a = 2$, $b = 3$

$P = 3^3 - 2^2 = 27 - 4 = 23$ is prime number

$Q = ba - ab = 3^2 - 2^3 = 9 - 8 = 1$ neither prime nor composite

60. **B, C**

1. $A = \pi r^2$; $\pi(2r)^2 = 4\pi r^2$ (False)

2. $A = \frac{1}{2}bh$; $\frac{1}{2} \times b \times 2h = bh$ (True)

3. $A = lb$; $(2l) \times b = 2lb$ (True)

61. **A, B** $a^2+b^2 = a^2 - i^2b^2 = a^2 - (ib)^2 = (a+ib)\ (a-ib)$

62. **B, C, D** 3, 4, 5; 6, 8, 10; 5, 12, 13

63. **A, C**

$2^{2x+3} - 9 \times 2^x + 1 = 0$; $\left(2^x\right)^2.2^3 - 9 \times 2^x + 1 = 0$

$8a^2 - 9a + 1 = 0$; $8a^2 - 8a - a - 1 = 0 \Rightarrow 8a(a-1) - 1(a-1) = 0$

$(a-1)(8a-1) = 0 \Rightarrow$ a = 1 or 1/8

$2^x = 1 = 2^0$ $2^x = 1/8 = 2^{-3}$

x = 0 x = -3

64. **A, B** Equilateral triangle, Square

65. **A, B, C** 2, 3; 3, 8; 5, 7

66. **625** The pattern is 1^0, 2^1, 3^2, 4^3, $\underline{5^4}$

67. **30** The pattern is 1x2, 2x3, 6x4, 24x5, 120x6

68. **225** All others are squares of prime numbers

69. **Wednesday**

70. **44** 22 in 12 hours $\therefore$ 44 in 24 hours

71. **35** Then years ago, let A's age is x; then B's is 2x

Present → A's age is x+10, B's is 2x+10

$\therefore \frac{x+10}{2x+10} = \frac{3}{4} \Rightarrow x = 5$

$\therefore$ Present A's age = 5+10=15; B's is 10+10=20

Total = 15 + 20 = 35 Years

72. **2 : 3** Let the ratio is x:1 $\therefore$ 16.4x + 15.4 = (x+1)15.8

$\Rightarrow$ 0.6 x = 0.4 $\Rightarrow$ x = 2/3 or x : 1 = 2 : 3

73. **15%** Let first discount be x%

Then 87.5 % of (100 – x)% of 160 is 119

$\Rightarrow \frac{87.5}{100} \times \frac{100-x}{100} \times 160 = 119 \Rightarrow 100 - x = 85$

$\therefore$ x = 15%

74. **18 years** Let sum = x; S.I. = x; Rate = $\frac{100 \times x}{x \times 6} = \frac{50}{3}\%$

Now sum = x; S.I. = 3x; Rate = $\frac{50}{3}\%$

$\therefore$ Time = $\frac{3x \times 100}{x \times 50/3} =$ 18 years

75. **8** Units digit in 781^{29} is 1; $(446)^{39}$ is 6; $(223)^{49}$ is 3

$\therefore$ 1 x 6 x 3 = 18 $\Rightarrow$ Unit's digit is 8.