

# KENNEDY MATHS OLYMPIAD - 2011 - SOLUTIONS - CLASS IX

## CLASS IX

01. A → greater than 2
02. D → 30%.  
It divided into five triangles each one 20%.  
one triangle + half triangle = 30%.
03. C →  $x + \frac{1}{x}$   

$$x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x} = (x + \frac{1}{x})^2 + 2 - 2(x + \frac{1}{x})$$

$$(x^2 - 2) + 2 - 2x \Rightarrow a^2 - 2a = a(a-2)$$

$$(x + \frac{1}{x})(x + \frac{1}{x} - 2)$$

 $x + \frac{1}{x} = a$   
 $x^2 + \frac{1}{x^2} = a^2 - 2$
04. B → 2  

$$\frac{(a+b)^2 + (a-b)^2}{a^2 + b^2} = \frac{2(a^2 + b^2)}{a^2 + b^2} = 2$$
05. A → 0  

$$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2 = (a+b)(a+b+c)$$

$$(a+b+c)c = a+b+c$$

$$m=1 \quad n=-1 \quad m+n=0$$
06. B → 2
07. A → 8%.  
 $12 + 13 + 15 = 40 \Rightarrow \frac{40}{500} \times 100 = 8$
08. D → 9 faces, 16 edges, 9 vertices
09. E →  $\frac{1}{2}(LA + LC)$  Print mistake
10. B → 40  

$$\frac{\text{Volume of cylinder}}{\text{Volume of coin}} = \frac{V}{\pi r^2 h}$$
11. B → Personally or through appointed investigation for a specific purpose.
12. B → 20-30
13. C → 30.48  
 Volume of cube - Volume of sphere.
14. A →  $3\pi r^2$
15. A →  $xy = 9z$   
 $a^x = c^y \Rightarrow a = c^{y/x} \mid a^z = c^3 \Rightarrow a = c^{3/z}$   
 $c^{y/x} = c^{3/z} \Rightarrow \frac{y}{x} = \frac{3}{z} \Rightarrow xy = 9z$
16. E → C →  $\frac{10^b}{n}$   
 $\log_{10} m = b - (\log_{10} n) \Rightarrow \log_{10} mn = b \quad mn = 10^b \Rightarrow m = \frac{10^b}{n}$
17. C → 100) only
18. D → -1, -4 put  $x = \pm 1$  and solve.

19.  $D \rightarrow \text{None}$

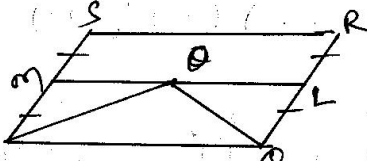
20.  $D \rightarrow \text{No solution}$  They are parallel lines

21.  $A \rightarrow a^3 I + 3a^2 b C$

22.  $D \rightarrow \text{memory}$

23.  $A \rightarrow 131$   $x$  and  $3x-16$   
 $x + 3x - 16 = 180 \Rightarrow x = 49 \therefore 3x - 16 = 131$

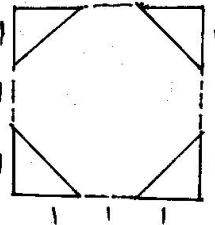
24.  $B \rightarrow \frac{1}{4} \times \text{or } 118m \text{ PQRS}$



$\Delta POQ = \frac{1}{2} 118m \text{ POLM}$   
 $= \frac{1}{2} \left( \frac{1}{2} 118m \text{ PQRS} \right) = \frac{1}{4} (118m \text{ PQRS})$

(25)  $C \rightarrow 7$

Square area - 4 triangles are  
 $= 3^2 - \frac{1}{2} \times 1 \times 1 \times 4$   
 $= 9 - 2 = 7$



(26)  $D \rightarrow \frac{3}{5}$  3:5

(27)  $B \rightarrow \frac{a-b}{a+b}$

$(x^2 - bx)(m+1) = (m-1)(cx - c)$   
 $(m+1)x^2 - b(m+1)x - (m-1)ax + c(m-1) = 0$   
 $(m+1)x^2 - [(m+1)b + (m-1)a]x + c(m-1) = 0$   
 $(m+1)b + (m-1)a = 0$   
 $mb + b + ma - a = 0$   
 $m(a+b) = a - b \Rightarrow m = \frac{a-b}{a+b}$

(28) add mark.

(29)  $C \rightarrow \frac{2abc}{a^2 + b^2 - ab}$

$\frac{1}{a} = \frac{1}{a} + \frac{1}{b}$ ,  $\frac{1}{b} = \frac{1}{a} + \frac{1}{b}$ ,  $\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$   
 $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \Rightarrow \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) - \left(\frac{1}{a} + \frac{1}{b}\right) = 0$   
 $= \frac{ab + bc + ca}{2abc} - \frac{1}{c} = 0$  and so on

(30)  $C \rightarrow 1$   $\left[ \frac{(a+1)^2 (a^2 - a + 1) (a+1)^2 (a^2 + a + 1)^2}{(a^3 + 1)^2 (a^3 - 1)^2} \right]^2 = \frac{(a^3 + 1)^2 (a^3 - 1)^2}{(a^3 + 1)^2 (a^3 - 1)^2} = 1$

31.  $D \rightarrow P \cap Q \cap R^c$

32.  $D \rightarrow 3$   $x + 6x = -\frac{(-14)}{p}$   $x \cdot 6x = 8/p$   
 $7x = \frac{14}{p}$   $p x^2 = \frac{8}{6} = \frac{4}{3}$   
 $p \Rightarrow x = \frac{2}{p}$   $p \cdot \frac{4}{p^2} = \frac{4}{3} \Rightarrow p = 3$

(33)  $C \rightarrow \text{Sum of reciprocals of } a, b \text{ is } 1$

(34)  $E \rightarrow \begin{bmatrix} -5 & 5 \\ -5 & -1 \end{bmatrix}$   $2(A - 2B) = \begin{bmatrix} 2 & -2 \\ 6 & 0 \end{bmatrix} = 2A - 4B$   
 $(2A - 3B) - (2A - 4B) = \begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} -5 & 5 \\ -5 & -1 \end{bmatrix}$

7)  $C \rightarrow$  parallelogram

(36)  $C \rightarrow -1/4$

$$7^{4x+3} - 30 = 7^{4x+1} + 18$$

$$7^{4x+1} = a \quad 499 - 30 = a + 18 \Rightarrow 48a = 48 \Rightarrow a = 1$$

$$7^{4x+1} = 7^0 \Rightarrow x = -1/4$$

(37)  $A \rightarrow 11$

$$\log x + 2 \log 3 = 2 \log(x-y) - \log y$$

$$\log 9x = \log \frac{(x-y)^2}{y}$$

$$x^2 + y^2 - 2xy = 9xy \Rightarrow x^2 + y^2 = 11xy$$

$$\frac{x^2 + y^2}{xy} = 11$$

(38)  $B \rightarrow am - 1/b$

Slope are equal (parallel lines)

(39)  $B \rightarrow (-1, 5)$

$\angle B = 90^\circ$   $\triangle ABC$  right triangle

(40)  $B \rightarrow 81$

$$\text{diagonal} = \sqrt{9+9} = 9\sqrt{2}$$

$$\text{Area} = \frac{d^2}{2} = \frac{81 \times 2}{2} = 81$$

(41)  $C \rightarrow 40$

$$x_n - x_1 = 8 \Rightarrow (5x_n + 7) - (5x_1 + 7) = 5(x_n - x_1) = 5 \times 8 = 40$$

(42)  $D \rightarrow 4$

$$k + (k+2) = b \quad | \quad k(k+2) = c$$
  
$$b = 2(k+1) \quad | \quad c = k(k+2)$$
  
$$b^2 - 4c = 4(k+1)^2 - 4k(k+2)$$
  
$$= 4k^2 + 8k + 4 - 4k^2 - 8k = 4$$

(43)  $C \rightarrow 0$

$$3^{68} \times 2^{73} \times 2^{73} \times 3^{101} \times 5^{101}$$

(44)  $C \rightarrow A$

$$AB = B \Rightarrow A(BA) = A$$
  
$$(AB)A = BA \Rightarrow A \cdot (BA) = A \Rightarrow A \cdot A = A \Rightarrow A^2 = A$$
  
$$A^{20} = A$$

(45)  $D \rightarrow$  Subset  $A \times B$

(46)  $C \rightarrow a = b$

(47)  $D \rightarrow c < 0$

(48)  $D \rightarrow B \& C$

(49)  $E$

$$\sqrt{50+548} = \sqrt{5\sqrt{2} + 2\sqrt{2}\sqrt{6}} = \sqrt{2} \sqrt{5 + 2\sqrt{2}\sqrt{3}}$$
  
$$= \sqrt{2} (\sqrt{2} + \sqrt{3})$$

(50)  $D \rightarrow 5$

$$1^{99} \Rightarrow 1 \quad | \quad 3^{99} \Rightarrow 7$$
  
$$2^{99} \Rightarrow 8 \quad | \quad 4^{99} \Rightarrow 4$$
  
$$5^{99} \Rightarrow 5$$
  
$$1 + 8 + 7 + 4 + 5 = 25$$

(51)  $C \rightarrow 3, 6, 2$

True & error method.

(52)  $D \rightarrow \frac{1}{2}$

$$\log_{16} [\log_{16} 4 + 3\frac{1}{2}] = \log_{16} [\frac{1}{2} + \frac{7}{2}] = \log_{16} 4 = \frac{1}{2}$$

(53)  $A \rightarrow B$  belongs

(54)  $E \rightarrow \begin{bmatrix} -1 & 1 \\ -1 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & B \\ \alpha & \alpha \end{bmatrix} \begin{bmatrix} \alpha & B \\ 1 & B \end{bmatrix} = \begin{bmatrix} \alpha+B & B+B^2 \\ \alpha^2+\alpha & \alpha B+\alpha B \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\begin{aligned} \alpha+B &= -1 \Rightarrow (B = -1-\alpha) / (B = -1-\alpha) \\ \alpha B &= 1 \Rightarrow \alpha(-1-\alpha) = 1 = -\alpha - \alpha^2 = 1 \Rightarrow \alpha + \alpha^2 = -1 \\ & \alpha + \alpha^2 = -1 \end{aligned}$$

(55)  $B \rightarrow B - A$

(56) A, B, E, D

57. A, B, C

58. A, C, D

59. A, C

60. A, B, C

61. A, C

62. A, B

63. A, B, C, D

64. C, D

65. B, D

66) 4.5 Terms are alternatively multiplied by 1.5 and divided by 3.

67) 91 (should be 90). Subtract squares of natural numbers from 1.

68) 276. Sum of first two digits is 2 more than the third digit in all other numbers

69) 197.5 Solution: Angle formed is 162.5

70) 2022.

71) 200 Solution: LCM of 2,4,5 = 20. Hence cube of  $20 \times 20 \times 20 = 8000$ . Divided by cuboid volume =  $8000 / 2 \times 4 \times 5 = 8000 / 40 = 200$

72) 30 Solution: Let A finishes the job in x days, then B does it in 3x days which means 2x days = 80 days. Therefore, A takes 40 days and B takes 120 days to finish the job. For one day, A does 1/40 and B does 1/120 of the job; together they will do  $1/40 + 1/120 = 1/30$ . Hence, for the total job, they take 30 days

73) 140.  $36+36+34+34$

74) 24.  $5x+3 : 4x+3 = 11x : 9x$  implies  $x=6$ . Therefore, B's age is  $4x=24$ .

75) 100